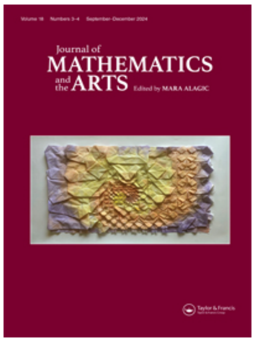

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Leonardo's Vitruvian Man: modern craniofacial anatomical analysis reveals a possible solution to the 500-year-old mystery

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Leonardo's Vitruvian Man: modern craniofacial anatomical analysis reveals a possible solution to the 500-year-old mystery

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ABSTRACT

For over 500 years, Leonardo da Vinci's geometric system for establishing the precise relationship between the circle and square in his Vitruvian Man drawing has remained a mystery. This paper demonstrates that Leonardo's explicit textual reference to 'an equilateral triangle' between the figure's legs provides his construction method and reveals the anatomical foundation for his proportional choices. The analysis shows that Leonardo's equilateral triangle corresponds to Bonwill's triangle in dental anatomy—the foundational geometric relationship governing optimal human jaw function. Leonardo's systematic construction yields a ratio of 1.64–1.65 between the square's side and circle's radius, matching both published measurements of the original drawing and the tetrahedral ratio of 1.633 found in optimal spatial arrangements. This ratio corresponds to modern calculations of optimal human craniofacial proportions (cranial architecture: 1.64 ± 0.04), suggesting Leonardo identified geometric principles of optimal spatial organization that contemporary science recognizes as important to human anatomical optimization. The findings position Vitruvian Man as both artistic masterpiece and prescient scientific hypothesis about the mathematical relationships governing ideal human proportional design.

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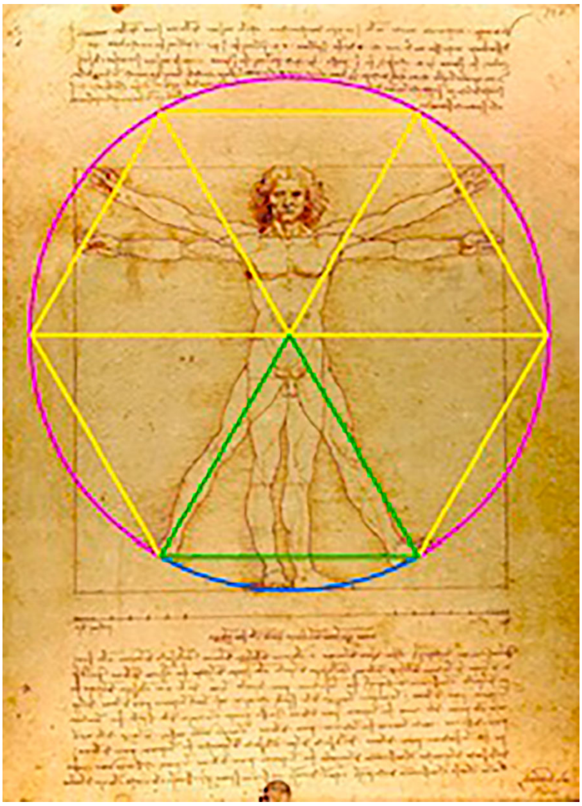


Highlights

- Leonardo's textual reference to 'an equilateral triangle' between the figure's legs provides insight into his geometric approach, offering a potential solution to the long-standing question of his circle-square relationship.
- Leonardo's equilateral triangle corresponds to Bonwill's triangle in dental anatomy – the geometric relationship governing optimal human jaw function established in 1864, suggesting Leonardo may have identified similar principles centuries earlier.
- The measured ratio of 1.64–1.65 in Leonardo's drawing approximates the tetrahedral ratio of 1.633 – a mathematical relationship that appears in optimal sphere packing, tensegrity structures, and Fuller's Vector Equilibrium.
- Modern anatomical measurements, including cranial architecture ratios of 1.64 ± 0.04 found exclusively in humans, suggest convergence around similar mathematical relationships across biological systems.
- The analysis shows how combining art historical scholarship with contemporary anatomical knowledge can illuminate sophisticated mathematical thinking underlying Renaissance artistic achievement.

1. Introduction: the geometric construction question

1.1. The geometric construction problem

Leonardo da Vinci's Vitruvian Man (c. 1490) represents one of the most analyzed yet cryptic works in art history. While the drawing's symbolic significance as representing ideal human proportions has been extensively documented, the precise geometric system Leonardo used to establish the relationship between the circumscribing circle and square has remained mysteriously elusive for over five centuries.

The challenge originates from Vitruvius himself, who proposed in 'De Architectura' that the perfectly proportioned human figure could be inscribed within both a circle and a square, but provided no mathematical framework for achieving this geometric relationship. As Murtinho (2015) documents, 'nowhere in the Vitruvian treaty is there a clarification of the proportional system that establishes the relational factor between the square and the circle. This situation has led to immense geometric and symbolic speculation in terms of the search for and definition of the rules that will have guided Leonardo to the drawing of his Vitruvian man.'

This geometric question has attracted scholarly investigation for centuries because Leonardo achieved what Vitruvius only proposed – a precise mathematical relationship that successfully inscribes the human figure within both geometric forms. Understanding Leonardo's construction method has implications beyond art history, potentially revealing sophisticated mathematical and anatomical insights embedded within Renaissance artistic practice.

1.2. Previous attempts at a solution

The search for Leonardo's geometric method has generated numerous theories, each attempting to explain the measured relationship between the circle and square in the original drawing. Scholarly measurements consistently report a ratio of approximately 1.64–1.65 between the square's side length (181.5 mm) and the circle's radius (110 mm) (Ida, 2012; Mascia, 2016; Murtinho, 2015). However, minor discrepancies highlight the methodological challenges in precisely measuring historical drawings, particularly given the subtle imperfections documented by Murtinho. Specifically, he noted that 'the square's vertical sides [are] slightly skewed' and the circle is 'built from successive arcs.' This observation about the imperfect alignment of the square is critical to understanding the range of measurements reported in the literature. Notably, Mascia (2016) suggests that the square may have been deliberately tilted to accommodate the dual postures depicted in the drawing.

The Golden Ratio Theory: The most popular explanation has been that Leonardo employed the Golden Ratio ($\varphi \approx 1.618$), particularly given his documented collaboration with mathematician Luca Pacioli on 'Divina Proportione.' However, as Murtinho demonstrates, this construction produces significant error – over 2% deviation from the actual measurements – which seems unlikely given Leonardo's documented geometric precision in other works.

Alternative Geometric Constructions: Other scholars have proposed various geometric systems. Helbing (2005) suggested a heptagon construction, while March (1998) proposed an octagon-based system that achieves closer approximation to the measured values.

Murtinho developed his own ‘double vesica piscis’ construction, which produces the smallest deviation (0.7 mm) from the original measurements.

The Fundamental Problem: While these geometric constructions can approximate Leonardo’s measurements, they suffer from a critical weakness: none explain why Leonardo would choose these specific proportions. They remain purely abstract mathematical exercises without connection to Leonardo’s documented interests in human anatomy, functional relationships, or natural principles. As geometric puzzles, they succeed; as explanations of Leonardo’s methodology and intentions, they fail to provide convincing rationale for his specific choices.

1.3. The missing key: Leonardo’s own words

The solution to this geometric mystery has been hiding in plain sight within Leonardo’s own manuscript notes accompanying the drawing. In his characteristic mirror script, Leonardo wrote: ‘If you open your legs enough that your head is lowered by one-fourteenth of your height and raise your hands enough that your extended fingers touch the line of the top of your head, know that the centre of the extended limbs will be the navel, and the space between the legs will be an equilateral triangle’ (Leonardo, 1970).

Mascia (2018) says ‘A geometric analysis carried out on three homonymous works, known as the Vitruvian man, by Leonardo da Vinci, Giacomo Andrea da Ferrara and Gulielmo Philandro has identified the equilateral triangle as a common feature.’ It is hypothesised that Leonardo’s equilateral triangle corresponds to a fundamental principle of human craniofacial architecture that wouldn’t be formally established until the 19th century.

2. The Solution: Leonardo’s Equilateral Triangle System

2.1. Bonwill’s triangle: the foundation of modern dental geometry

The key to understanding Leonardo’s geometric system emerges from recognizing the anatomical significance of his equilateral triangle reference. In 1864, dentist William Bonwill established that optimal human mandibular function is based on an equilateral triangle connecting the two mandibular condyles (jaw joints) to the midpoint of the lower central incisors (Figure 1). Like the Vitruvian ideal, this precise morphotype is not a commonly observed phenomenon (Nikolopoulou et al., 2019).

Bonwill’s triangle represents more than an arbitrary geometric construction – it defines optimal functional relationships within the human craniofacial complex. Each side of this equilateral triangle measures approximately 4 in. and this triangular framework governs ideal tooth positioning, jaw relationships, and mandibular movements during function. Modern dental science has validated Bonwill’s discovery, with contemporary dental measurement instruments (articulators) being based on it (Ganesh & Mohanraj, 2022).

The significance of Bonwill’s equilateral triangle extends beyond dental function to encompass broader principles of human facial architecture. Von Spee identified that the biting surfaces of the lower teeth align along a curved arc via the temporomandibular joint (TMJ). This curve forms part of a circle with a radius of approximately 101.6 mm – the

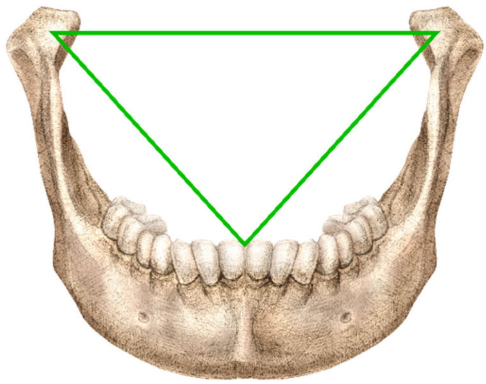


Figure 1. Bonwill's Triangle in Mandibular Anatomy. Bonwill's equilateral triangle in mandibular anatomy, demonstrating the foundational geometric relationship governing optimal human jaw function established in 1864. This triangle connects the two mandibular condyles (jaw joints) to the midpoint of the lower central incisors and corresponds precisely to Leonardo's explicit reference to 'an equilateral triangle' in his Vitruvian Man construction.

same measurement as the sides of Bonwill's triangle. Modern three-dimensional analyses have confirmed this circular arrangement, which enables optimal occlusal contact during mandibular movement (Kumar & Tamizharasi, 2012).

Monson's Spherical Theory (1920): Monson expanded these observations into three dimensions, proposing that all teeth lie along the surface of an imaginary sphere with its centre near the forehead (glabella) and a radius of approximately 4 in. (101.6 mm). Within this configuration, the mandible forms a tetrahedron with Bonwill's equilateral triangle as its base and its apex extending to the glabella (Figure 2).

Recent analysis by Loto (2017; 2018) unified several dental geometries into what he referred to as the tetrahedral theory of occlusion. An analysis of his work reveals a striking mathematical relationship: When Monson's sphere has a radius of 101.6 mm, the circumsphere of the tetrahedron formed by connecting 4 Bonwill's triangles has a radius of approximately 62.2 mm. The ratio between these spheres equals $101.6/62.2 \approx 1.633$.

Leonardo's construction demonstrates a parallel relationship to Loto's dental analysis. Just as Loto's ratio compares the static structural foundation of the jaws (Bonwill's tetrahedron circumsphere) to the dynamic functional envelope (Monson's sphere) that encompasses mandibular movement, Leonardo's measured ratio relates the figure's static positioning to its dynamic capability. The square contains the figure in static cruciform pose – arms horizontal and legs together – while the circle encompasses the figure with arms raised and legs spread to form the equilateral triangle. This geometric relationship between static form and dynamic potential mirrors the structural-functional relationships that govern optimal craniofacial architecture.

The convergence of Leonardo's geometric insights with modern dental architecture studies suggests these systems may be solving for a fundamental structural principle that wasn't formally recognized until the twentieth century. Buckminster Fuller (1961) introduced the concept of 'tensegrity' – describing structural systems as 'islands of compression in an ocean of tension' that achieve maximum strength with minimum material

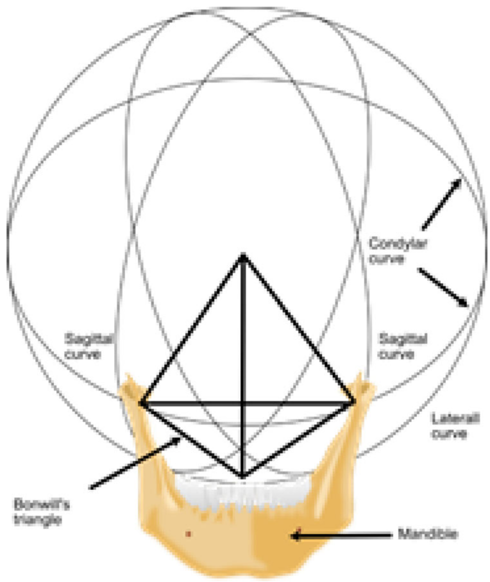


Figure 2. Monson's Spherical Theory and Tetrahedral Configuration. Monson's spherical theory and tetrahedral configuration showing how the mandible forms a tetrahedron with Bonwill's equilateral triangle as its base and apex extending to the glabella. This three-dimensional dental geometry demonstrates the tetrahedral relationships that yield the 1.633 ratio between Monson's sphere and the circumsphere of the tetrahedral unit.

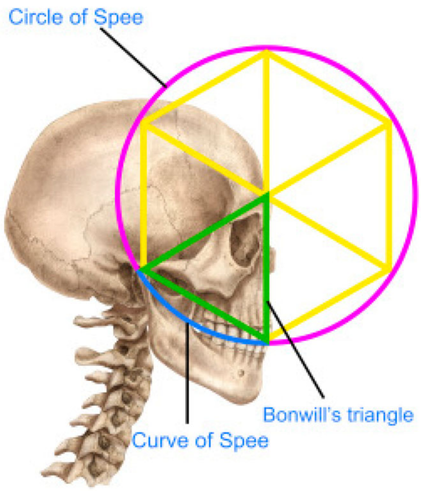
through optimal force distribution rather than rigid connections. When viewed through this tensegrity framework, both Leonardo's 1.64–1.65 ratio in Vitruvian Man and Loto's documentation of tetrahedral relationships in dental occlusion appear to address the same fundamental challenge: how biological systems optimize force distribution through geometric form.

The dental studies reveal that human jaw architecture naturally organizes around tetrahedral and triangular geometries that maximize mechanical efficiency during mastication – precisely the configurations that tensegrity principles would predict for optimal force transmission (Figure 3a). Similarly, Leonardo's geometric construction may have intuited the proportional relationships that optimize the body's capacity to distribute mechanical loads through its fascial and musculoskeletal networks (Figure 3b).

3. The tetrahedral ratio: mathematical foundation

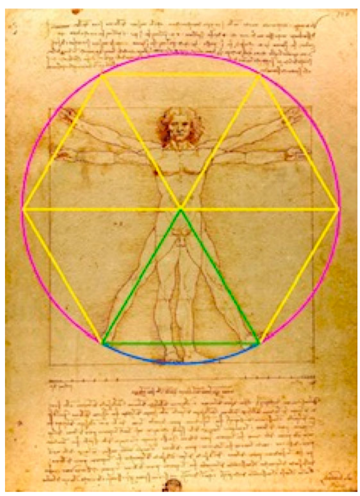
3.1. Derivation and significance

The tetrahedral ratio of 1.633 emerges from the fundamental geometry of the regular tetrahedron – the simplest three-dimensional polyhedron that can maintain structural integrity through pure tension and compression forces. This analysis identifies this ratio as appearing consistently across optimal spatial arrangements. In a regular tetrahedron with edge length 'a', the height (h) from any vertex to the centre of the opposite face equals $a\cdot(\sqrt{6})/3$. The ratio between the edge length (a) and the twice the height (2h) can be



(a)

Figure 3a. Tetrahedral Geometry in Human Craniofacial Architecture. The human skull demonstrates natural organization around tetrahedral and triangular geometries that maximize mechanical efficiency. Bonwill's triangle (green) forms an equilateral triangle connecting the two mandibular condyles and the midpoint of the lower central incisors. The Circle of Spee (purple) and Curve of Spee (blue) represent optimal arrangements for dental occlusion that naturally emerge from these tetrahedral relationships. This geometric organization maximizes force transmission efficiency during mastication, revealing the tetrahedral ratio's presence in human anatomical optimization.



(b)

Figure 3b. Leonardo's Vitruvian Man and Optimal Geometric Relationships. Leonardo's construction demonstrates the same tetrahedral principles found in craniofacial architecture. The equilateral triangle (green) that Leonardo explicitly referenced between the figure's legs, when replicated six times around the navel, creates the hexagonal pattern (yellow) that generates the measured ratio of approximately 1.64 between the square's side and circle's radius. This geometric relationship may represent Leonardo's intuitive recognition.

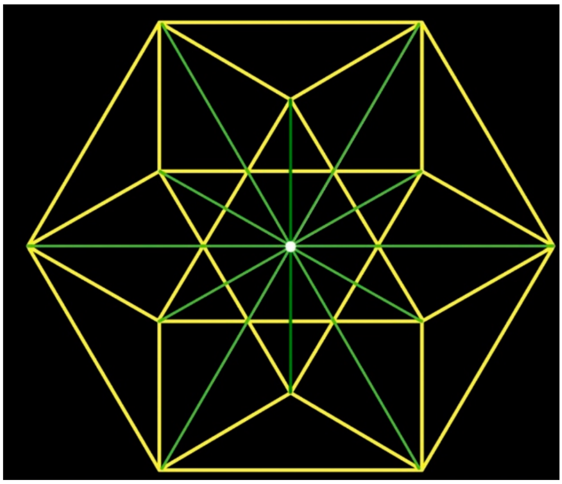


Figure 4. Fuller's Vector Equilibrium Structure. Fuller's Vector Equilibrium formed by four hexagons intersecting at 60°, demonstrating the fundamental geometric structure of perfect tensegrity. This configuration represents the uniquely balanced form where vector forces from the center point are perfectly balanced by circumferentially restraining vectors, creating the geometric foundation underlying optimal spatial organization.

expressed as:

$$a : 2h = a : [(2a \cdot \sqrt{}6)/3] = 3 : 2\sqrt{6} \approx 1 : 1.633.$$

This value – 1.633 – represents a geometric ratio that emerges specifically from tetrahedral geometry. It defines a specific proportional relationship that appears in optimal spatial arrangements, particularly in the geometry of sphere packing and tensegrity structures.

3.2. Vector Equilibrium

The tetrahedral ratio of 1.633 emerges naturally when eight tetrahedrons are arranged around a common centre, creating a configuration of perfect geometric balance. Fuller discovered this uniquely balanced geometric form in 1917 and named it the Vector Equilibrium (VE) in 1940 (Fuller, 1975). In this structure, all vectors maintain equal length: the 24 edges connecting the circumferential vertices and the 12 omnidirectional vectors extending at 60° from the centre point. This equality creates perfect equilibrium, with vector forces from the centre point balanced by the circumferentially restraining vectors (Figure 4).

The VE can also be constructed from four hexagons intersecting at 60°. Because the surface pattern repeats, the VE can be spherically tiled by making a stereographic projection of four great circles (circles with a plane that intersects the centre of a sphere). To form these circles, straight lines on the polyhedron are projected as continuous arcs on a plane.

The connection between the VE and optimal sphere packing reveals the mathematical foundation underlying both Leonardo's construction and biological architecture. The fundamental unit of optimal sphere packing is a tetrahedron – when four spheres are arranged so that each touches the other three, their centres naturally form the vertices of a regular tetrahedron. When expanded to include a central sphere surrounded by twelve spheres in closest contact, this arrangement creates the VE configuration with one sphere at the center and the remaining twelve spheres occupying the vertices of the surrounding polyhedron.

3.3. Empirical validation

In hexagonal close packing of spheres – the most efficient three-dimensional arrangement – the ratio of the unit cell height (c) to the basal plane dimension (a) equals $\sqrt{(8/3)} = 1.633$ (Hales, 2006). This empirical ratio emerges directly from the geometric constraints of optimal sphere packing (Figure 5). Crystallographically, this same ratio governs the structure of numerous compounds that naturally adopt hexagonal close-packed arrangements, including zinc, cadmium, and magnesium (Kittel, 2005), confirming the tetrahedral ratio as a fundamental principle of optimal spatial organization throughout nature.

A study by Tamargo and Pindrik (2019) examined 100 human skulls and identified a consistent ratio of 1.64 ± 0.04 in cranial architecture, measuring the nasioiniac arc (curved distance from nasion to inion along the skull surface) against the parieto-occipital arc (curved distance from bregma to inion). This ratio appears exclusively in humans and represents optimal structural relationships within the skull that closely approximate the tetrahedral ratio.

Loto's geometric analysis provides additional empirical validation through his calculation that the maximum number of Monson's pyramids (tetrahedra based on Bonwill's triangle) that can fit within Monson's sphere is exactly 4. This constraint emerges from the mathematical relationship between the tetrahedron's circumsphere and the containing sphere – when optimally packed, four tetrahedral units create a perfect tetrahedral arrangement within the larger sphere. This tetrahedral packing principle reflects the same geometric constraints that govern optimal sphere packing and the VE configuration, suggesting that biological systems follow universal geometric laws of optimal spatial organization.

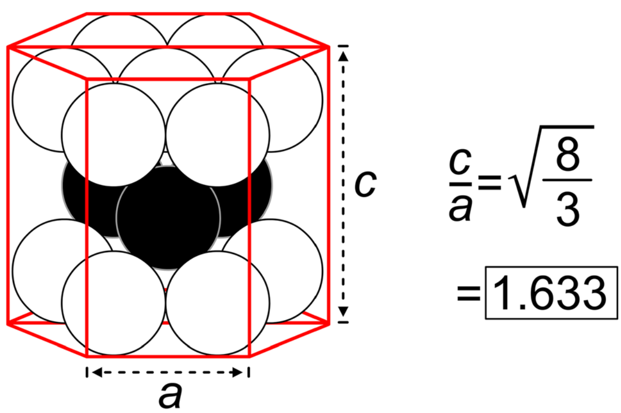
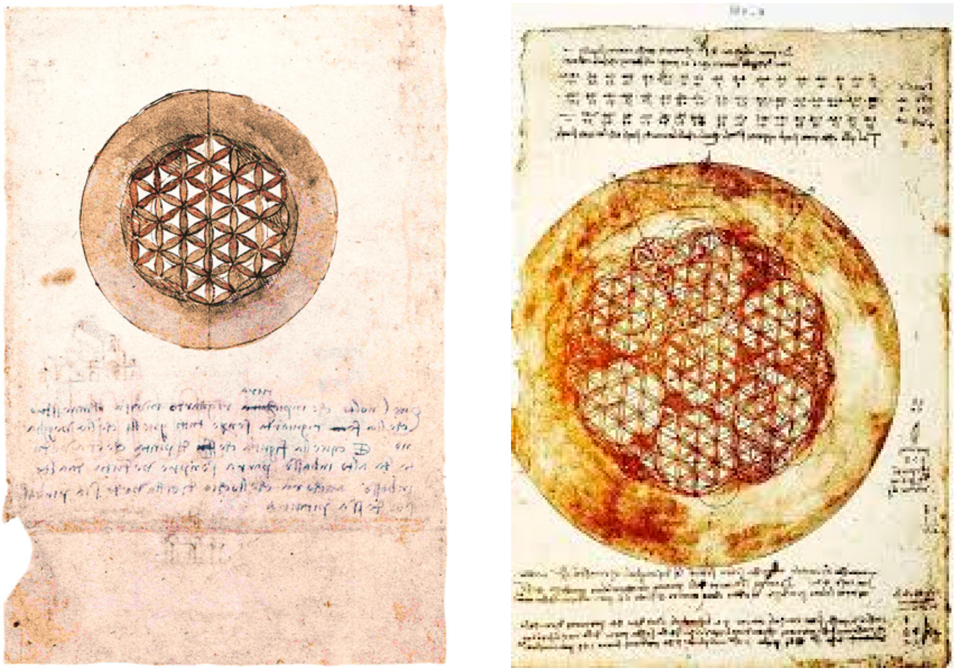


Figure 5. Hexagonal Close Packing of Spheres. Hexagonal close packing of spheres demonstrating the mathematical foundation underlying Leonardo's geometric system. The ratio $c/a = \sqrt{(8/3)} = 1.633$ represents optimal three-dimensional spatial organization and corresponds closely to the proportional relationships Leonardo achieved through his equilateral triangle construction, revealing a tetrahedral ratio that emerges from efficient spatial arrangement.



(a) (b)

Figure 6a,b. Leonardo's Hexagonal-Circle Pattern Studies. Leonardo's hexagonal-circle pattern studies from the Codex Atlanticus (folios 307v and 309v), demonstrating his systematic investigation into overlapping circular arrangements that create triangular tessellations. These drawings provide direct evidence that Leonardo was actively exploring the same principles of efficient spatial packing that modern mathematics recognizes as fundamental to optimal organization, supporting the argument that his Vitruvian Man construction was based on sophisticated geometric understanding.

Leonardo's own geometric studies provide direct evidence of his systematic investigation into these optimal spatial arrangements. His hexagonal circle pattern studies in the Codex Atlanticus (Figure 6a and 6b) demonstrate his detailed exploration of overlapping circular arrangements that create triangular tessellations – precisely the geometric relationships that naturally generate the previously termed tetrahedral ratio. These drawings show Leonardo actively investigating the same principles of efficient spatial packing that modern mathematics recognizes as fundamental to optimal organization, providing visual proof that his approach to the Vitruvian Man was grounded in sophisticated geometric understanding rather than arbitrary artistic choices.

The convergence of multiple independent measurements around the 1.633–1.64 range provides compelling evidence for the biological significance of these geometric relationships:

- Hexagonal close packing of spheres: c/a ratio = 1.633 (theoretical optimum)
- Human cranial architecture: 1.64 ± 0.04 (Tamargo & Pindrik, 2019)
- Dental tetrahedral theory: 1.633 (Loto's analysis of Monson's sphere relationships)
- Leonardo's Vitruvian Man: 1.64–1.65 (measured by multiple scholars)

This consistency across theoretical mathematics, biological structures, and Leonardo's artistic construction suggests that the previously termed 1.633 constant represents a fundamental ratio governing optimal spatial organization in both synthetic and biological systems.

4. Discussion: convergent evidence

The evidence examined in this paper suggests that Leonardo's Vitruvian Man incorporates geometric principles that anticipate modern understanding of optimal biological architecture. The measured proportions between the circle and square, yielding ratios of 1.64–1.65, bear striking similarity to the previously termed tetrahedral constant a mathematical relationship that defines optimal spatial arrangements in both synthetic and biological systems.

Leonardo's explicit reference to the equilateral triangle between the figure's legs provides the key to understanding his geometric construction method. This triangle corresponds to Bonwill's triangle in dental anatomy, establishing the anatomical foundation for his proportional choices. The geometric relationships embedded in the drawing reflect principles of tensegrity and optimal force distribution that wouldn't be formally recognized until the twentieth century.

The subsequent discovery of similar ratios in multiple domains validates Leonardo's vision: hexagonal close-packed crystal structures exhibit the ideal c/a ratio of 1.633, human cranial architecture shows ratios of 1.64 ± 0.04 found exclusively in humans, and human dental geometry reveals relationships involving optimal triangular and spherical arrangements.

5. Conclusion

This analysis reveals that Leonardo's Vitruvian Man incorporates geometric principles that anticipate modern understanding of optimal biological architecture. The convergence of Leonardo's measured ratios (1.64–1.65) with the previously termed tetrahedral constant suggests that human anatomy has evolved according to geometric principles that govern optimal spatial organization throughout the universe. From the hexagonal close packing of atoms to the tetrahedral architecture of dental occlusion to the cranial proportions found exclusively in humans, we observe consistent mathematical relationships encoded in biological form.

Leonardo's geometric construction successfully encoded fundamental spatial relationships in human form, demonstrating the remarkable precision of his Renaissance vision of mathematical unity between the human figure and natural order. Vitruvian Man stands as a testament to Leonardo's insight that human proportions reflect deeper mathematical principles governing optimal spatial organization.

Fuller's Isotropic Vector Matrix – essentially a fractal extension of the VE repeated throughout all space (Fuller, 1975) – provides a compelling framework for understanding these relationships, suggesting that space itself may be organized according to tetrahedral-octahedral geometries that generate the previously termed tetrahedral ratio. The geometric duality embedded in Leonardo's construction reveals a fundamental relationship: the triangular components – exemplified by Leonardo's explicit equilateral triangle and Bonwill's triangle – naturally tessellate into spherical arrangements through optimal packing principles, generating the circular boundary. Simultaneously, the octahedral relationships within Fuller's space-filling matrix create the square geometry - the VE itself displays prominent square faces where octahedral elements intersect (Figure 4). This triangular-to-circular and octahedral-to-square correspondence suggests why Leonardo's circle-to-square ratio appears to encode the previosuly termed tetrahedral constant.

In this context, Leonardo's cosmographia del minor mondo – his vision that the human body reflects universal mathematical principles – finds remarkable validation. The same geometric relationships that appear in optimal crystal structures, biological architectures, and Fuller's coordinate systems seem to be encoded in human proportions, suggesting that Leonardo intuited fundamental truths about the mathematical nature of reality itself.

This discovery positions Leonardo as both anatomical investigator and geometric philosopher – someone who identified optimal human proportional relationships four centuries before modern science would validate his insights, revealing sophisticated mathematical thinking underlying Renaissance artistic achievement.

The implications for dental science are particularly profound. While Vitruvian Man has long been questioned as a credible anatomical diagram, this research reveals that Leonardo encoded the precise mathematical relationships that govern optimal human craniofacial function. The correspondence between Leonardo's 1.64–1.65 ratios and the tetrahedral ratio of 1.633—the same ratio that defines optimal sphere packing and appears in human cranial architecture exclusively—suggests Vitruvian Man represents legitimate anatomical optimization rather than idealized artistic proportion.

This mathematical validation opens new possibilities for dental science. If human craniofacial systems have indeed evolved according to the same geometric principles that govern optimal spatial organization throughout nature, our understanding of dental function, treatment planning, and prosthetic design could be fundamentally transformed. Rather than viewing dental anatomy as arbitrary biological form, we might approach it as an expression of universal mathematical principles of optimal spatial efficiency—principles that Leonardo intuited centuries before modern science could validate them.

Declarations

The author declares no financial, commercial, or conflict of interest in relation to this research. Generative AI, specifically Claude (developed by Anthropic), was used as a research assistant tool for reviewing literature, analyzing geometric principles, drafting figure descriptions, and refining the manuscript text. All scientific interpretations, theoretical frameworks, and conclusions were conceived, developed, and validated by the human author.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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